Untangling the Knots: A Procedure for Identifying Discernibility Conflicts on a Cartographic Line

Barry J. Kronenfeld, Barbara P. Buttenfield, Lawrence V. Stanislawski and Ethan Shavers

ABSTRACT: Reducing detail on polyline features aids in legibility, making features appear more distinct and preventing coalescence with other features. Current metrics for evaluating generalization outcomes emphasize geometric change rather than legibility. The present study reports on development and testing of a vector-based metric of the discernibility of a single polyline feature or group of features, defined as the absence of visual coalescence at a target map scale. This metric prioritizes legibility problems due to resolution and the physical and optical limits of discernibility. The metric identifies specific locations of coalescence and is invariant to translation and rotation, providing a consistent measure across display contexts. A procedure for computing the above definition of discernibility and identifying the locations of discernibility conflicts will be presented. The algorithm is currently being tested in Python code, with a goal to include this tool in an open-source toolbox for cartographic generalization assessment.

KEYWORDS: *cartographic generalization, discernibility, legibility, scale reduction, assessment metrics*

Introduction

Cartographers have long performed simplification and smoothing operations on polyline features to produce maps at a reduced scale. In addition to reducing the number of feature vertices, it has been argued that eliminating detail on polyline features aids in legibility, making features appear more distinct and preventing different portions of a feature from overprinting or coalescing. On the other hand, metrics used to assess the quality of generalized linear cartographic features have primarily focused on positional displacement and distortion of morphometric properties such as length and sinuosity. Although it is important to minimize error and distortion, such metrics do not capture the underlying motivation of producing a more legible map feature in which each portion of the polyline can be discerned from every other.

Interest has recently increased in assessing the legibility of line features on reduced-scale maps to support automated cartographic generalization. Touya et al. (2016) use "legibility" to encompass multiple aspects of map readability, including an overall lack of complexity, entropy, and clutter. However, there is a distinction between difficulties in the cognitive processing of

map information that arise from too many symbols or symbols that are poorly organized, and difficulties in the visual perception of map symbols due to resolution and the physical or optical limits of discernibility. Focusing on the latter, Cheng et al. (2021) propose a Degree of Legibility (DoL) metric derived by rasterizing a polyline and then counting the proportion of rasterized pixels that are not involved in a topologically modified structure due to visual coalescence. Although the DoL metric shows promise in identifying illegible portions of a polyline, it is subject to variability under translation and rotation of the input polyline. It also fails to capture the precise degree and direction of coalescence at specific locations along the polyline.

The present study aims to develop a vector-based metric of the discernibility of a single polyline feature or group of features, defined as the absence of visual coalescence at a target map scale. Like the DoL, the proposed discernibility metric may facilitate the identification of polylines that require generalization and the evaluation of simplification and smoothing algorithm outputs (Cheng et al. 2021). In contrast to the raster-based DoL, whose value depends on a reference raster grid's specific origin and orientation, the proposed metric is invariant to translation and rotation, providing a consistent measure across display contexts. In computing a single numerical measure of overall discernibility, specific locations of coalescence will also be identified. These locations may serve as a basis for further cartographic generalization, such as exaggeration and pruning.

Concepts and Metric Definition

A starting point for conceptualizing such a vector-based discernibility metric is Perkal's (1966a, 1966b) analogy of a ball rolling along the polyline, with the diameter of the ball equal to the minimum discernible gap between line features (Figure 1). The outer trace of the rolling ball forms an outline similar to the result of a polyline buffer operation. Locations where this trace self-intersects are considered *discernibility conflicts* (Figure 1a).

Although the trace of Perkal's rolling ball captures something essential about the discernibility of a polyline, it is not workable in practice. Consider two consecutive line segments on a polyline that form a sharp protruding angle (Figure 1b). Such a protrusion will present a challenge to discernibility, as the gap between portions of the polyline approaching the protruding angle will be smaller than the minimum discernible gap, indicated by the crossing dashed red lines in Figure 1b. However, the standard geographic information system (GIS) buffer operation does not capture this crossing. Conceptually, this may be interpreted as the buffer overlapping itself. It is tempting to modify Perkal's rolling ball analogy to handle this situation, for example, by extending segments of the buffer outline that meet at an acute angle (illustrated by the extended dashed red lines in Figure 1). But what if the angle is not so acute (Figure 1c)? How should the metric handle configurations involving multiple line segments within the region where the buffer overlaps itself (Figure 1d)? These questions illustrate the need for a semantic framework to clarify the meaning of discernibility, one that can handle all such cases in a logical and consistent manner.



Figure 1: Illustration of polylines with potential discernibility conflicts. Dashed red lines represent path traced by a rolling ball with diameter equal to half the minimum discernible gap between lines at target map scale.

We propose that a discernibility conflict involves pairs of points that both (1) have a gap between them that is smaller than the minimum discernible distance at target map scale, and (2) are located far enough from each other along the path of the polyline that they should be discernable. Formally, discernibility of any two points a and b is defined by a function of the following form:

 $discernibility_{a,b} = f(g_{a,b}, \ell_{a,b})$

where $g_{a,b}$ denotes the size of the gap between a and b, and $\ell_{a,b}$ denotes the length of the polyline segment extending from a to b. The gap size is a function of the generalization scale and display type. Naturally, this function should have a higher value (i.e., be more discernible) when $g_{a,b}$ is larger and $\ell_{a,b}$ is smaller. Further, discernibility should be high when $g_{a,b}$ is above a certain threshold no matter the value of $\ell_{a,b}$. In other words, the function should follow the illustration in Figure 2.



Figure 2: Regions of discernibility and indiscernibility based on the gap between two points a & b $(g_{a,b})$ and the length of the polyline section between them $(\ell_{a,b})$.

An overall index of discernibility for a set of polylines can be defined as the proportion of points within a set that are discernible from every other point.

Implementation

A procedure for computing the above definition of discernibility and identifying the locations of discernibility conflicts will be presented. Because the number of pairs of points along a polyline is technically infinite, the first step is to prove that all discernibility conflicts can be identified via a finite iteration over the vertices of the polyline. This proof is constructed by analyzing the properties of $g_{a,b}$ and the ratio of $g_{a,b}$ to $\ell_{a,b}$ in terms of the medial axis of adjacent polygons constructed on either side of the analyzed polyline. Specifically, any point with a discernibility conflict with another point must either be part of a connected set of points that includes a vertex of the analyzed polyline(s) or conflict with another point that is part of such a connected set. This leads to the following general algorithm:

- 1. Loop through all vertices and determine whether they are involved in a discernibility conflict.
- 2. Loop through the resulting set of conflict vertices and traverse the polyline away from each conflict vertex in both directions until reaching the limits of the connected set of points involved in that conflict.

This algorithm is being tested in detail and will be implemented in Python code. We plan to include this code in an open-source Python toolbox for cartographic generalization assessment.

Disclaimer: Any use of trade, firm, or product names is for descriptive purposes only and does not imply endorsement by the U.S. Government.

References

- Cheng, X., Liu, Z. and Zhang, Q. 2021. MSLF: multi-scale legibility function to estimate the legible scale of individual line features. Cartography and Geographic Information Science, 48(2):151-168.
- Touya, G., Hoarau, C., and Christophe, S. 2016. Clutter and map legibility in automated cartography: A research agenda. Cartographica, 51(4):198-207. DOI:10.3138/cart.51.4.3132 (<u>https://hal.science/hal-02130692</u>)
- Perkal, J. 1966a. On the length of empirical curves. Ann Arbor, Mich.: *Michigan Inter-University Community of Mathematical Geographers*, Discussion Paper #10: 320-354. <u>https://deepblue.lib.umich.edu/handle/2027.42/58252</u>
- Perkal, J. 1966b. An attempt at objective line generalization. Ann Arbor, Mich.: *Michigan Inter-University Community of Mathematical Geographers*, Discussion Paper #10: 355- 373. <u>https://deepblue.lib.umich.edu/handle/2027.42/58252</u>

Barry Kronenfeld, Professor, Geology and Geography Department, Eastern Illinois University, Charleston, IL 61920

Barbara P. Buttenfield, Professor Emerita, Department of Geography, University of Colorado-Boulder, Boulder, CO 80303

Lawrence V. Stanislawski, Research Scientist, U.S. Geological Survey, Center of Excellence for Geospatial Information Science, Rolla, MO 65401

Ethan Shavers, Supervisory Geographer, U.S. Geological Survey, Center of Excellence for Geospatial Information Science, Rolla, MO 65401